

# Molecular views on thermo-osmotic flows

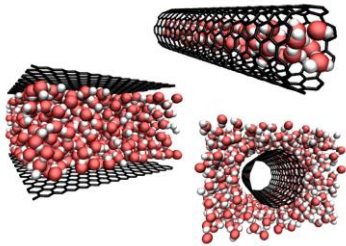
Li Fu, Samy Merabia, Laurent Joly

Univ Lyon, Université Claude Bernard Lyon 1, CNRS,  
Institut Lumière Matière, F-69622, Villeurbanne, France

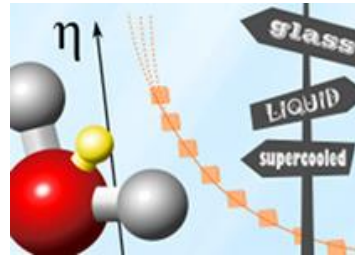
<http://ilm-perso.univ-lyon1.fr/~ljoly/>



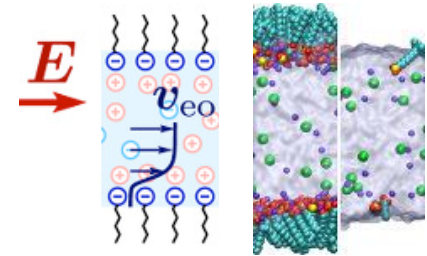
## Micro/nanofluidic transport: from molecular mechanisms to applications



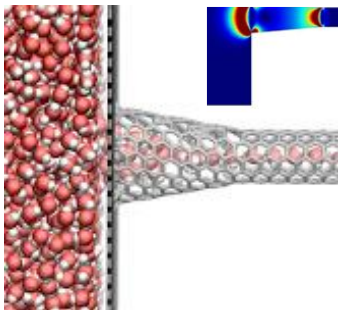
Interfacial hydrodynamics  
Liquid/solid friction



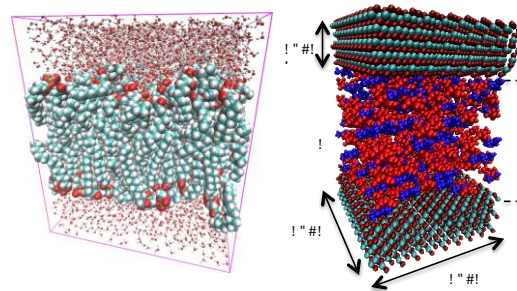
Understanding water



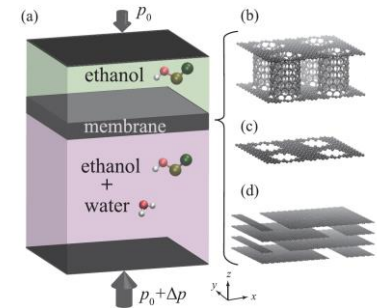
Energy conversion  
Electrokinetic effects



Nanoscale flows

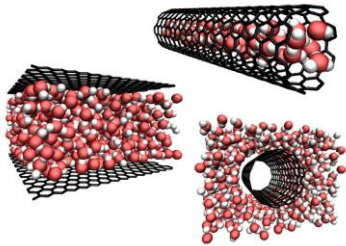


Lubrication

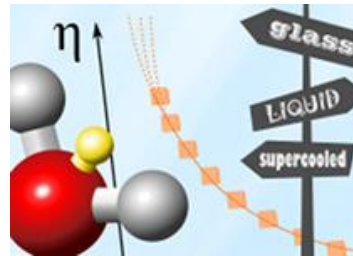


Devices

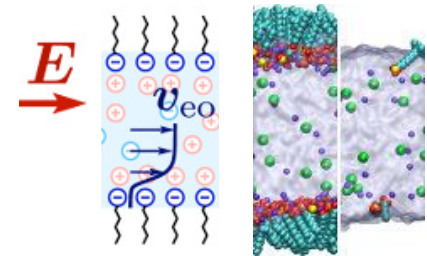
## Micro/nanofluidic transport: from molecular mechanisms to applications



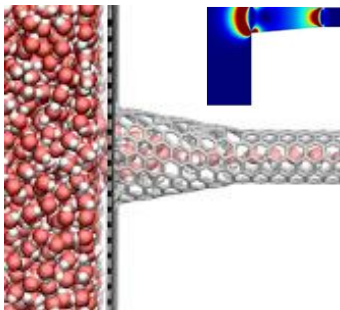
Interfacial hydrodynamics  
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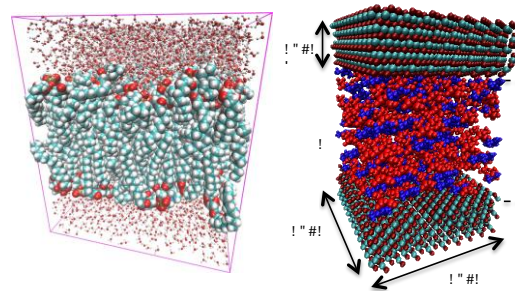
Understanding water



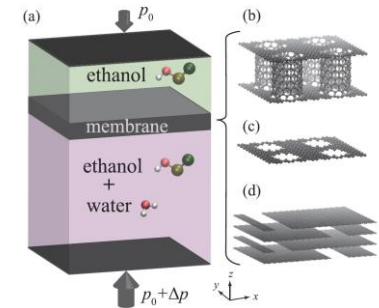
Energy conversion  
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Nanoscale flows



Lubrication

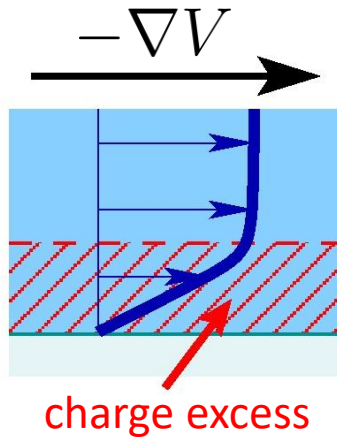


Devices

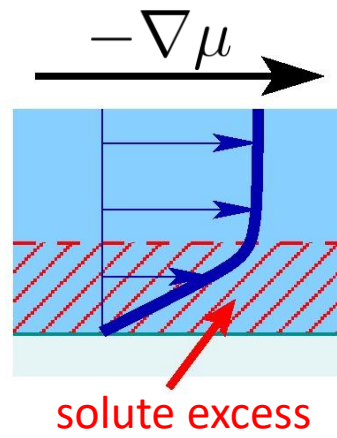
# Osmotic flows

Surface-driven flows generated by non-hydrodynamic forcing

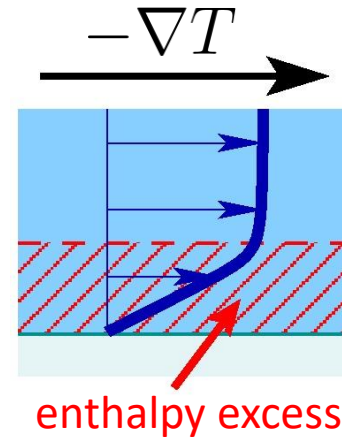
**Electro-osmosis**



**Diffusio-osmosis**



**Thermo-osmosis**



➔ **Micro/nanofluidics, sustainable energies (e.g. waste heat harvesting)**

Originate in a nanometric interfacial layer

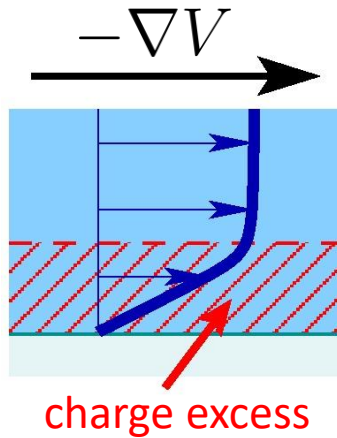


**Nanoscale structure and dynamics**

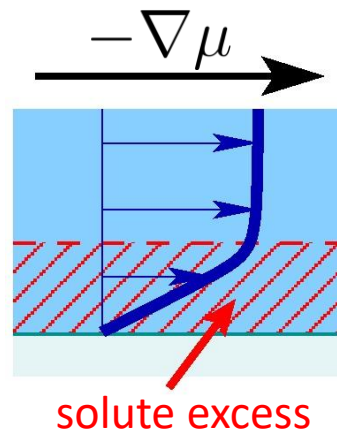
# Osmotic flows

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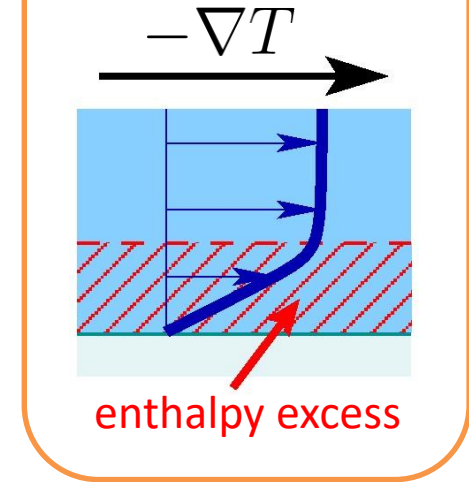
**Electro-osmosis**



**Diffusio-osmosis**



**Thermo-osmosis**



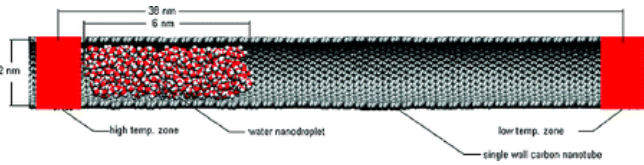
➔ **Micro/nanofluidics, sustainable energies (e.g. waste heat harvesting)**

Originate in a nanometric interfacial layer

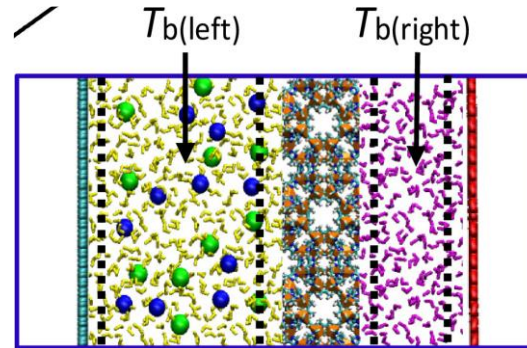


**Nanoscale structure and dynamics**

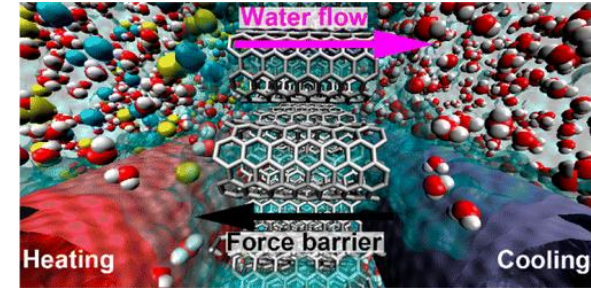
## For applications



Zambrano et al. Nano Lett 2009

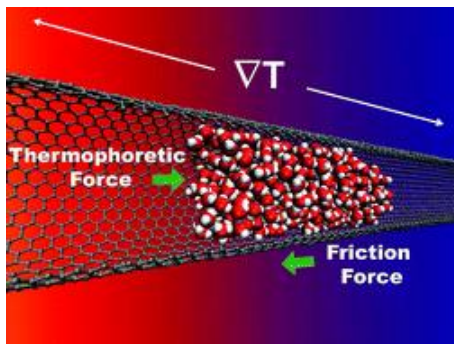


Gupta & Jiang ChemistrySelect 2017

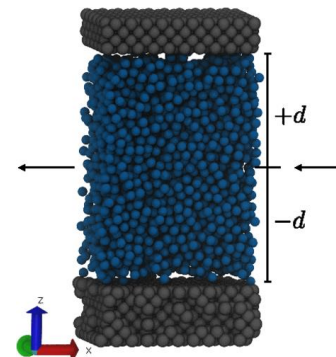


Zhao & Wu Nano Lett 2015

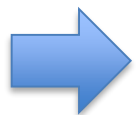
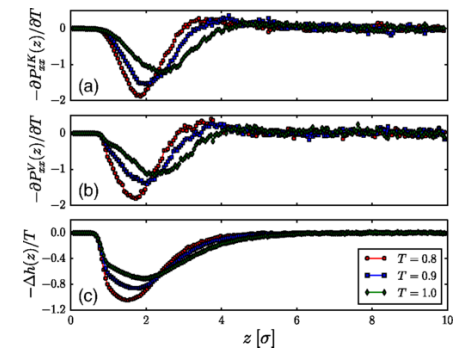
## Molecular mechanisms?



Oyarzua, Walther & Zambrano, PCCP 2018



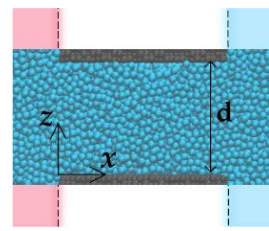
Ganti, Liu & Frenkel, PRL 2017



*Effect of wetting and interfacial hydrodynamics?*

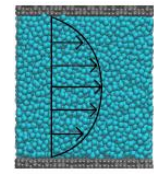
## Thermo-osmosis

➤ flow induced by a thermal gradient:  $v_s = M_{12} \left( -\frac{\nabla T}{T} \right)$




## Mechano-caloric effect

➤ heat flux induced by a pressure gradient:  $j_h = M_{21} (-\nabla p)$



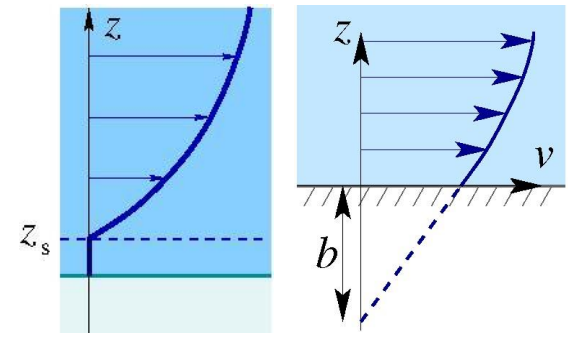
## Standard prediction:

$$M_{12} = M_{21} = \frac{1}{\eta} \int_0^{+\infty} z \delta h(z) dz \quad \text{Derjaguin and Sidorenkov, 1941}$$


enthalpy density excess

## Interfacial hydrodynamics: stagnant layer ( $z_s$ ), slip ( $b$ )...

$$M_{12} = M_{21} = \frac{1}{\eta} \int_{z_s}^{+\infty} (z - z_s + b) \delta h(z) dz$$

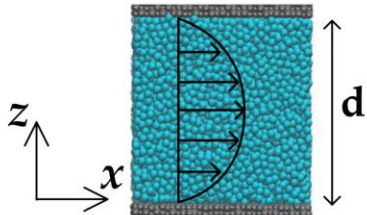


Following analogous treatment of electro-osmosis by Huang et al.

Generic Lennard-Jones interaction potential:  $V(r) = 4\epsilon[(\sigma/r)^{12} - (\sigma/r)^6]$

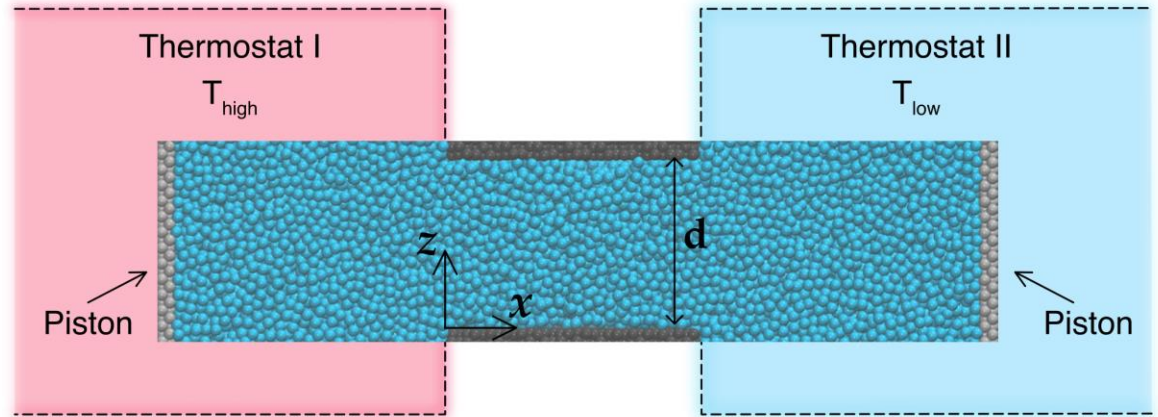
(a) Mechano-caloric configuration

External particle force  $f_i \rightarrow$



$$\delta j_h = M_{21}(-\nabla p)$$

(b) Thermo-osmotic configuration

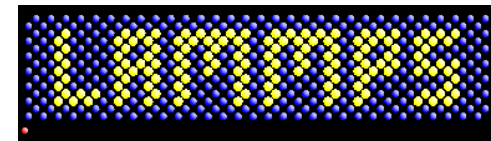


$$v_s = M_{12}\left(-\frac{\nabla T}{T}\right)$$

*Effect of wetting?*



**Varying liquid-solid interaction energy  $\epsilon_{ls}$**





## Measurements

$$j_h = M_{21}(-\nabla p)$$

$$j_h = \int \delta h(z) v_x(z) dz$$

$$-\nabla p = f_V = f_i N/V$$

$$v_s = M_{12}\left(-\frac{\nabla T}{T}\right)$$

from # of particles in reservoirs

from temperature profile in channel

## Theoretical prediction

$$M_{12} = M_{21} = \frac{1}{\eta} \int_{z_s}^{+\infty} (z - z_s - b) \delta h(z) dz$$

- **hydrodynamics**: from Poiseuille flow in mechano-caloric configuration
- **thermodynamics**: from equilibrium simulations

$$\delta h_{\text{eq}}(r; T) = h_{\text{eq}}(r; T) - h_{\text{bulk}}(T)$$

$$h(r) = (u_i(r) + p_i^z z(r)) \rho(r),$$

## Measurements

$$j_h = M_{21}(-\nabla p)$$

$$j_h = \int \delta h(z) v_x(z) dz$$

$$-\nabla p = f_V = f_i N/V$$

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## Theoretical prediction

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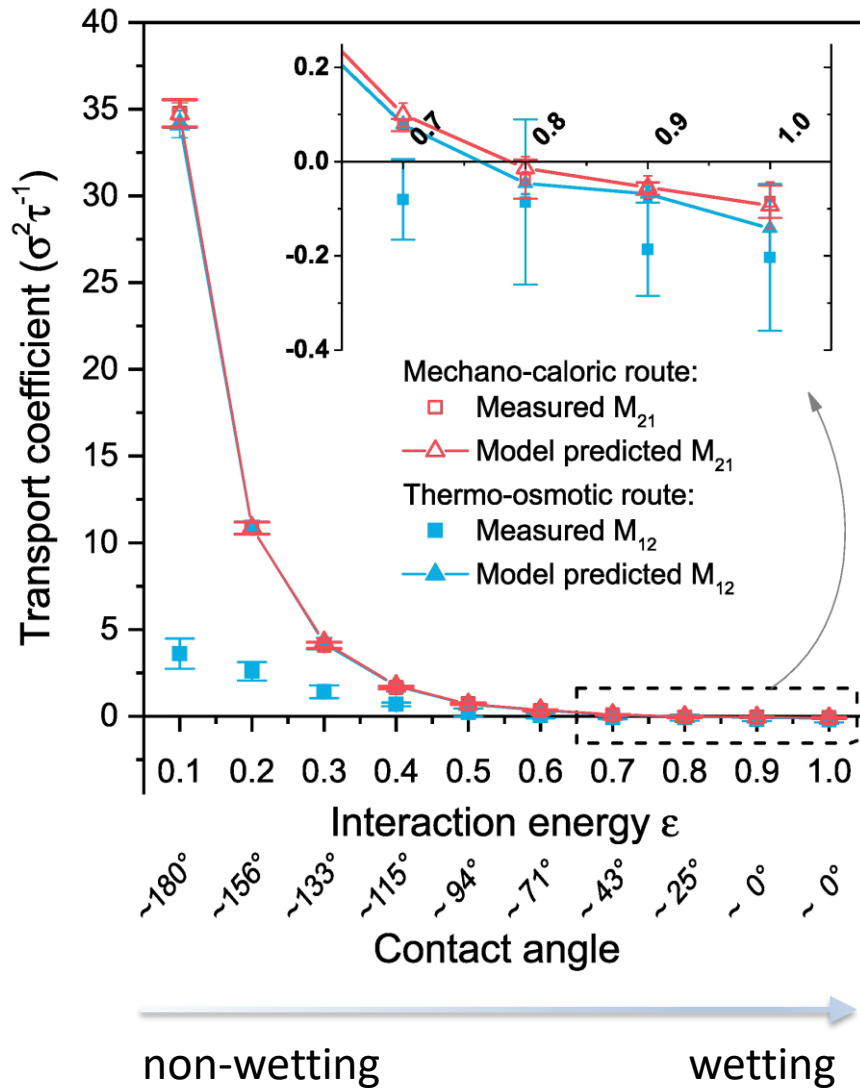
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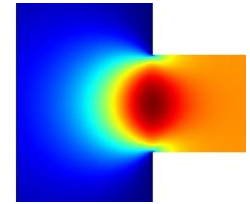
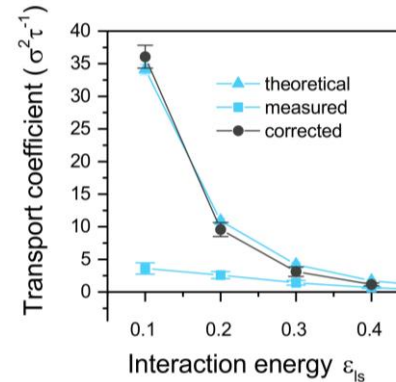
 compute stress/atom:

$$S_{ab} = - \left[ mv_a v_b + \frac{1}{2} \sum_{n=1}^{N_p} (r_{1a} F_{1b} + r_{2a} F_{2b}) + \frac{1}{2} \sum_{n=1}^{N_b} (r_{1a} F_{1b} + r_{2a} F_{2b}) + \frac{1}{3} \sum_{n=1}^{N_a} (r_{1a} F_{1b} + r_{2a} F_{2b} + r_{3a} F_{3b}) + \frac{1}{4} \sum_{n=1}^{N_d} (r_{1a} F_{1b} + r_{2a} F_{2b} + r_{3a} F_{3b} + r_{4a} F_{4b}) + \frac{1}{4} \sum_{n=1}^{N_i} (r_{1a} F_{1b} + r_{2a} F_{2b} + r_{3a} F_{3b} + r_{4a} F_{4b}) + \text{Kspace}(r_{ia}, F_{ib}) + \sum_{n=1}^{N_f} r_{ia} F_{ib} \right]$$



## Thermo-osmotic route

- viscous entrance effects



## Wetting systems

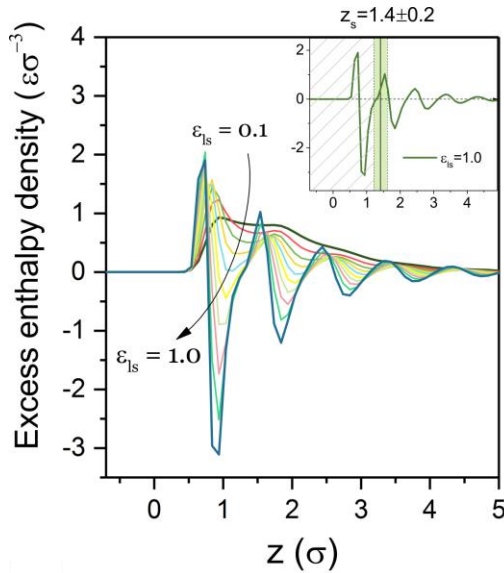
- $|M_{21}| \sim 0.1\sigma^2/\tau \sim 10^{-8} \text{ m}^2/\text{s}$
- cf. experiments  $\sim 10^{-10}$  to  $10^{-9} \text{ m}^2/\text{s}$
- **change of sign**

cf. Lüsebrink, Yang, Ripoll, JPCM 2012

## Non-wetting systems

- **giant response**

up to  $\sim 35\sigma^2/\tau \sim 4 \times 10^{-6} \text{ m}^2/\text{s}$

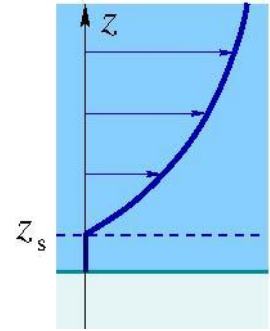


## Wetting systems

Oscillations of the excess enthalpy

- crucial role of  $z_s$

$$M_{12} = M_{21} = \frac{1}{\eta} \int_{z_s}^{+\infty} (z - z_s + b) \delta h(z) dz$$



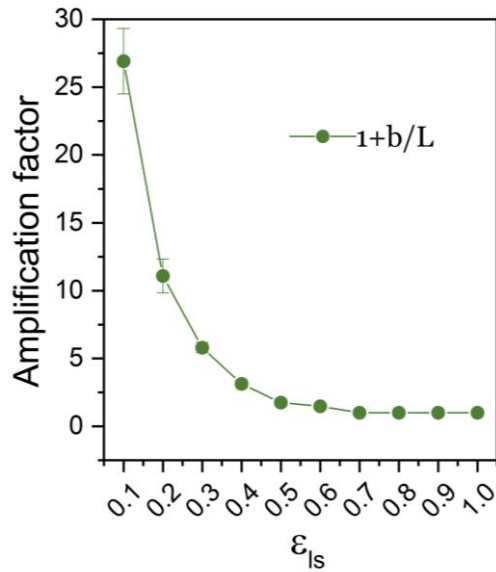
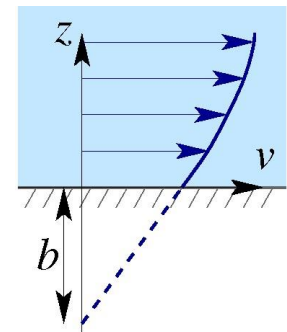
## Non-wetting systems

Molecular thickness of the interaction layer

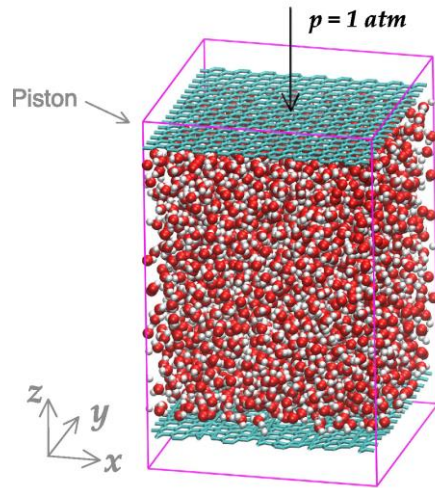
- large amplification by slip

cf.

- electro-osmosis: Joly et al. PRL 2004, JCP 2006
- Joly et al. PRL 2014, Barbosa De Lima & Joly 2017
- diffusio-osmosis: Ajdari & Bocquet, PRL 2006
- thermo-phoresis: Morthomas, Würger, JPCM 2009



# Water-graphene interface

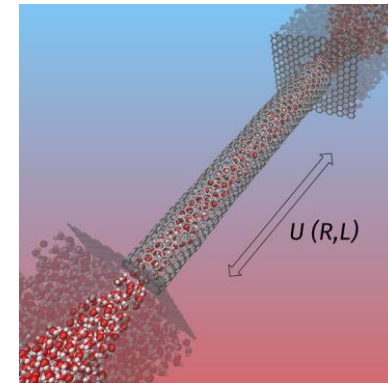


**Giant thermo-osmotic response:**

$$M_{21} = (2.5 \pm 0.3) \times 10^{-6} \text{ m}^2/\text{s}$$

## Waste heat harvesting with graphitic membranes?

- entrance pressure drop
- thermal short-circuit by walls
- desalination applications: ultra-confinement



## Analytical model coupling thermodynamics and hydrodynamics

$$U = \frac{R \overline{\delta h}}{2\lambda L + \pi C \eta} \times \frac{\Delta T}{T_{\text{avg}}} \times \left( 1 - \frac{T_{\text{avg}} \Delta \Pi}{\Delta T \overline{\delta h}} \right)$$

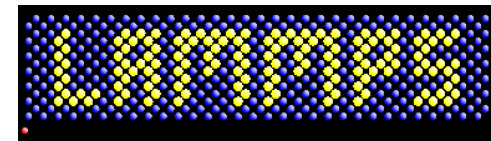
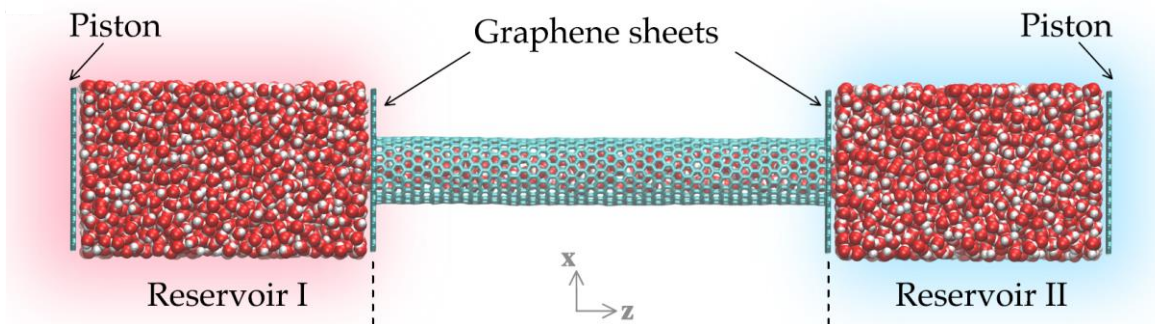
$\overline{\delta h}$  : average enthalpy excess density over the section of the tube

$\lambda$  : friction coefficient

$C$  : numerical constant depending on the geometry and the hydrodynamic boundary condition => entrance effects

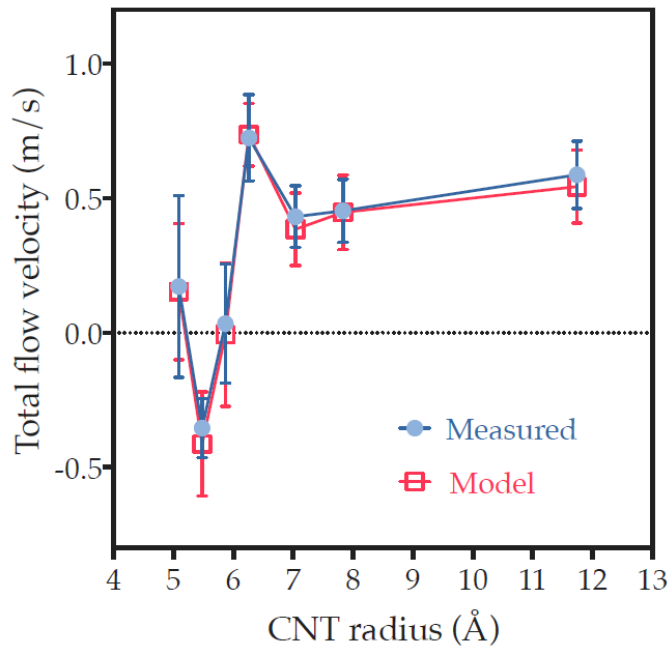
$\Delta \Pi$  : osmotic pressure

## MD simulations



## Pumping ( $\Delta\Pi = 0$ )

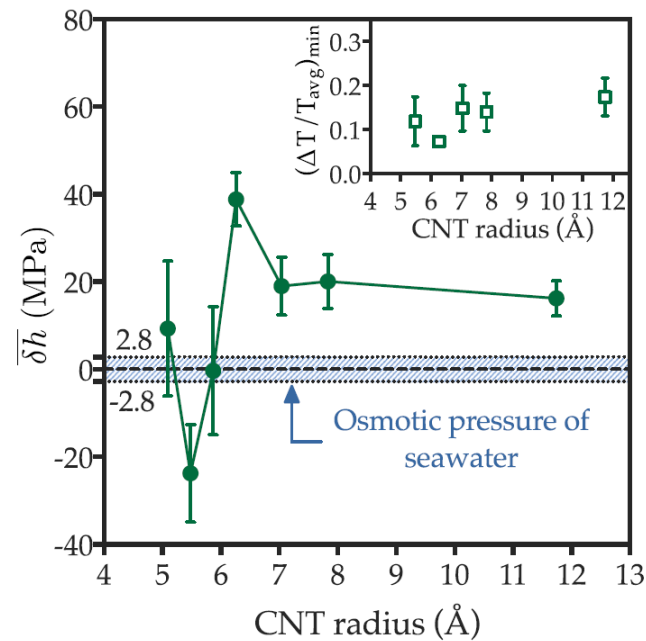
$$U = \frac{R\overline{\delta h}}{2\lambda L + \pi C\eta} \times \frac{\Delta T}{T_{\text{avg}}}$$



## Desalination / overcoming a pressure difference

$$\Delta\Pi_{\text{max}} = \overline{\delta h} \times \Delta T / T_{\text{avg}}$$

cf. Dariel & Kedem J. Phys. Chem. 1975



- **Model validated by MD results -> can help searching for other innovative membranes**
- **Fast and robust flows (even when extrapolated to experimental parameters)**

# Conclusion

## Osmotic flows generated in the nanometric vicinity of interfaces

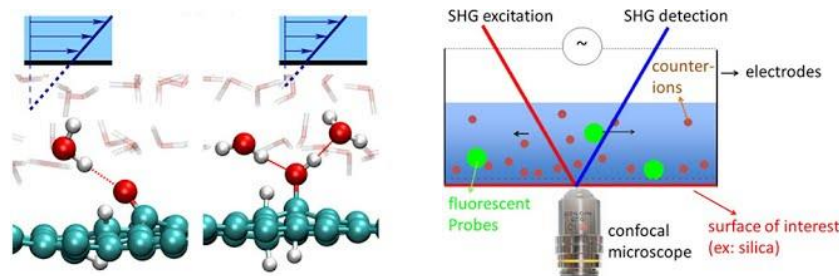
- Molecular **structure** and **dynamics**
- *Interfacial hydrodynamics can control amplitude and sign of the response!*

## ➡ Molecular dynamics simulations

However:

- Numerical investigations limited to classical molecular dynamics
- ➔ reactivity?  
see e.g. Joly et al., J. Phys. Chem. Lett. 2016

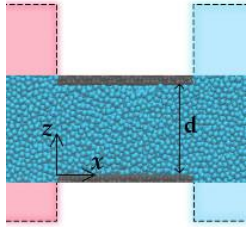
## ➡ NEctAR project: Nanofluidic Energy Conversion using reActive suRfaces <https://sites.google.com/site/anrnectar/>



Bonhomme, Blanc, Joly, Ybert, Bianco, Adv. Colloid Interface Sci. 2017

Hartkamp, Bianco, Fu, Dufrêche, Bonhomme, Joly, submitted to Curr. Opin. Colloid Interface Sci





Li Fu  
Samy Merabia

- French National Research Agency, project NEcAR
- Institut Universitaire de France

Joly, Detcheverry, Biance, PRL 2014

Barbosa De Lima, Joly, Soft Matter 2017

Lee, Cottin-Bizonne, Fulcrand, Joly, Ybert, J. Phys. Chem. Lett. 2017

Fu, Merabia, Joly, PRL 2017

Fu, Merabia, Joly, J. Phys. Chem. Lett. 2018

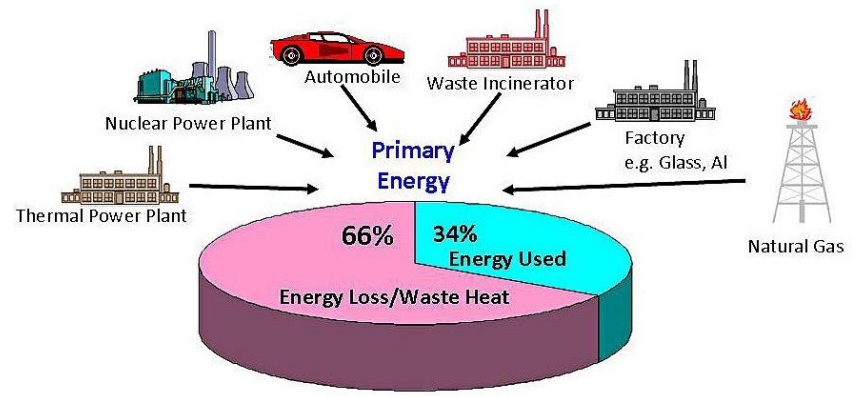
<http://ilm-perso.univ-lyon1.fr/~ljoly/>



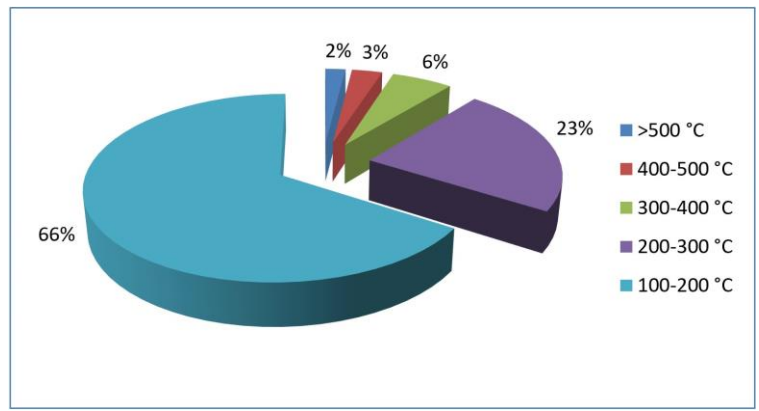


## Heat management: a crucial challenge

- Waste heat produced by industry and energy conversion
- Large fraction of low grade waste heat



Energy loss through waste heat  
(Penn State University)



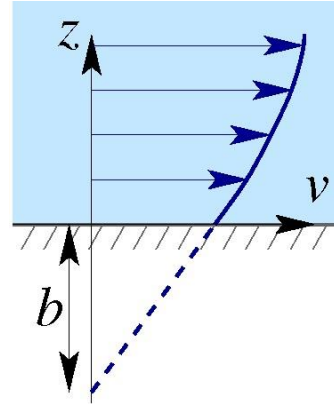
Distribution of waste heat in industry



*Could nanofluidic systems be used for (low grade) waste heat harvesting?*

## Hydrodynamic slip

$$v_{\text{slip}} = b \left. \frac{\partial v}{\partial z} \right|_{\text{wall}}$$



Slip length

$b \sim$  tens of nm

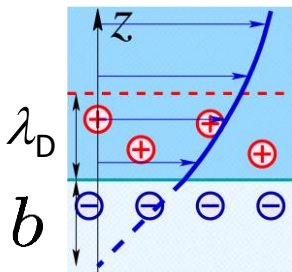


Flow amplification factor:

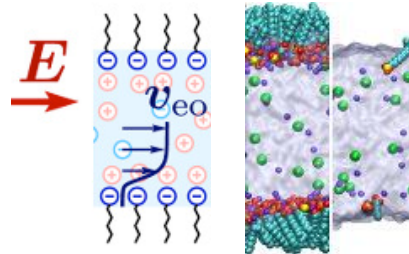
$$1 + \frac{b}{L}$$

interfacial layer thickness

## Electro-osmosis

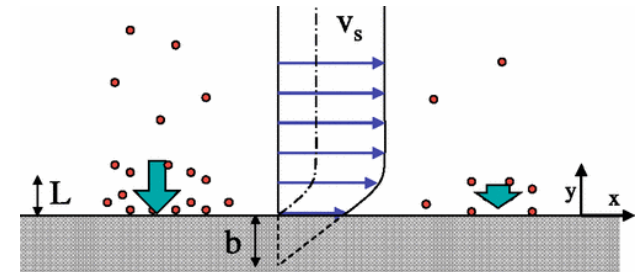


Joly et al., PRL 2004, JCP 2006



Joly et al., PRL 2014  
Barbosa De Lima & Joly 2017

## Diffusio-osmosis



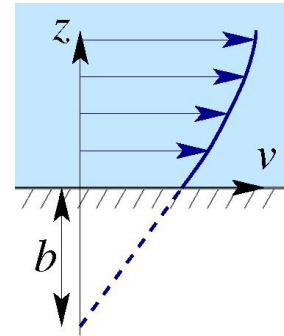
Ajdari & Bocquet, PRL 2006

## Continuum hydrodynamics (cst $\eta$ ), partial slip BC

$$\frac{d^2 v_x}{dz^2} = -\frac{E_x}{\eta} \rho_e(z) \quad \text{with} \quad v_x(z_w) = b \left. \frac{dv_x}{dz} \right|_{z=z_w}$$

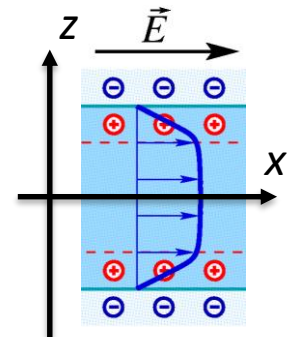
Stokes equation

Partial slip BC at  $z = z_w$

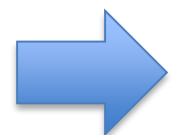


$$\zeta = \frac{\eta v_{eo}}{\varepsilon E_x} = \frac{\eta v_x(0)}{\varepsilon E_x} = \frac{1}{\varepsilon} \int_{z_w}^0 dz (z - z_w + b) \rho_e(z)$$

$\varepsilon$  = bulk permittivity, no assumption on  $\varepsilon(z)$  close to the surface



Uncharged surfaces:  $\int_{z_w}^0 dz \rho_e(z) = 0$



$$\zeta = \frac{1}{\varepsilon} \int_{z_w}^0 dz z \rho_e(z)$$

➤ **independent of  $b$**   
(i.e. of friction)

**Thermal gradient → thermodynamic force density**

$$f_{\text{thermo}} \approx \delta h_{\text{eq}}(r; T_{\text{avg}}) \frac{\nabla T(z)}{T_{\text{avg}}}, \quad \text{with} \quad \begin{aligned} \delta h_{\text{eq}}(r; T) &= h_{\text{eq}}(r; T) - h_{\text{bulk}}(T) \\ h(r) &= (u_i(r) + p_i^{zz}(r))\rho(r), \end{aligned}$$

**Fluid friction on the wall:**  $F_{\text{friction}} = 2\pi RL\lambda v_{\text{osm}}$

➤ **Bare thermo-osmotic velocity in the tube (large slip limit)**

$$v_{\text{osm}} = \frac{R \overline{\delta h}}{2\lambda L} \times \frac{\Delta T}{T_{\text{avg}}}. \quad \text{with} \quad \overline{\delta h} = \frac{1}{\pi R^2} \int_0^R dr 2\pi r \delta h_{\text{eq}}(r; T_{\text{avg}}),$$

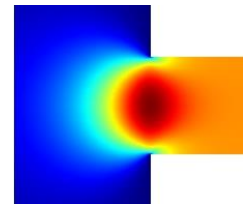
➤ **General hydrodynamic boundary condition**

$$v_{\text{osm}} = \frac{R^2}{4\eta L} \left(1 + \frac{2b}{R}\right) \times \overline{\delta h} \times \frac{\Delta T}{T_{\text{avg}}}. \quad \text{with} \quad \overline{\delta h} = \frac{1}{\pi R^2} \int_0^R dr 2\pi r \delta h \left(1 - \frac{r^2}{R^2 + 2bR}\right),$$

# Model (2)

## Backflow due to:

- viscous entrance pressure drop  $\Delta p_{\text{in}} = (\pi C \eta / R) \times U$
- pressure difference between reservoirs  $\Delta \Pi$



Sampson, 1891

- Total velocity through the membrane (large slip limit)**

$$U = \frac{R \bar{\delta h}}{2\lambda L + \pi C \eta} \times \frac{\Delta T}{T_{\text{avg}}} \times \left( 1 - \frac{T_{\text{avg}} \Delta \Pi}{\Delta T \bar{\delta h}} \right).$$

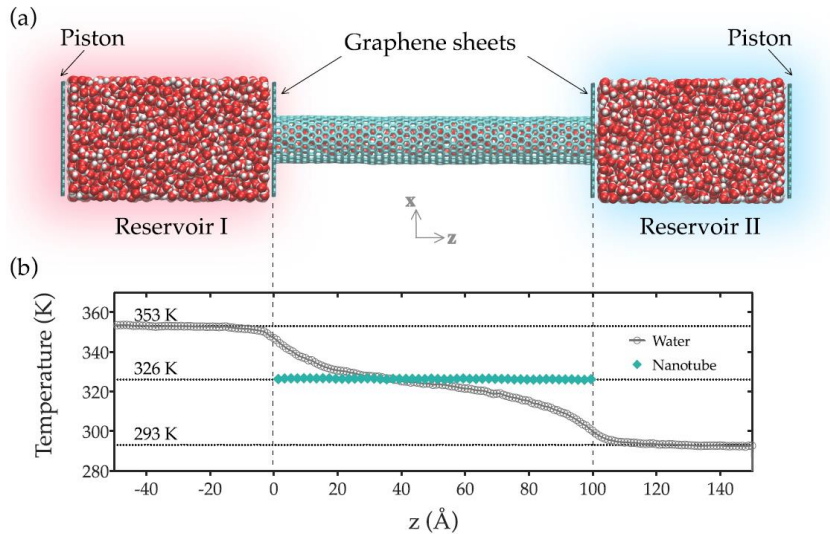
$$\Delta \Pi_{\text{max}} = \bar{\delta h} \times \Delta T / T_{\text{avg}}.$$

cf. Dariel & Kedem J. Phys. Chem. 1975

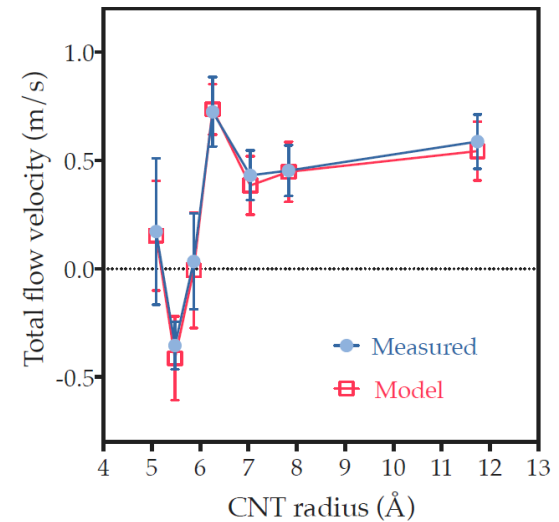
- General hydrodynamic boundary condition**

$$U = \frac{R^2 \left(1 + \frac{2b}{R}\right) \times \bar{\delta h} \times (\Delta T / T_{\text{avg}})}{4\eta L + \frac{\pi C \eta R}{2} \left(1 + \frac{4b}{R}\right)} \times \left( 1 - \frac{\left(1 + \frac{4b}{R}\right)}{2 \left(1 + \frac{2b}{R}\right)} \times \frac{T_{\text{avg}} \Delta \Pi}{\Delta T \bar{\delta h}} \right).$$

# Molecular dynamics



$$\text{Model } (\Delta\Pi = 0): U = \frac{R \overline{\delta h}}{2\lambda L + \pi C \eta} \times \frac{\Delta T}{T_{\text{avg}}}$$



## Quantitative match between model and MD results

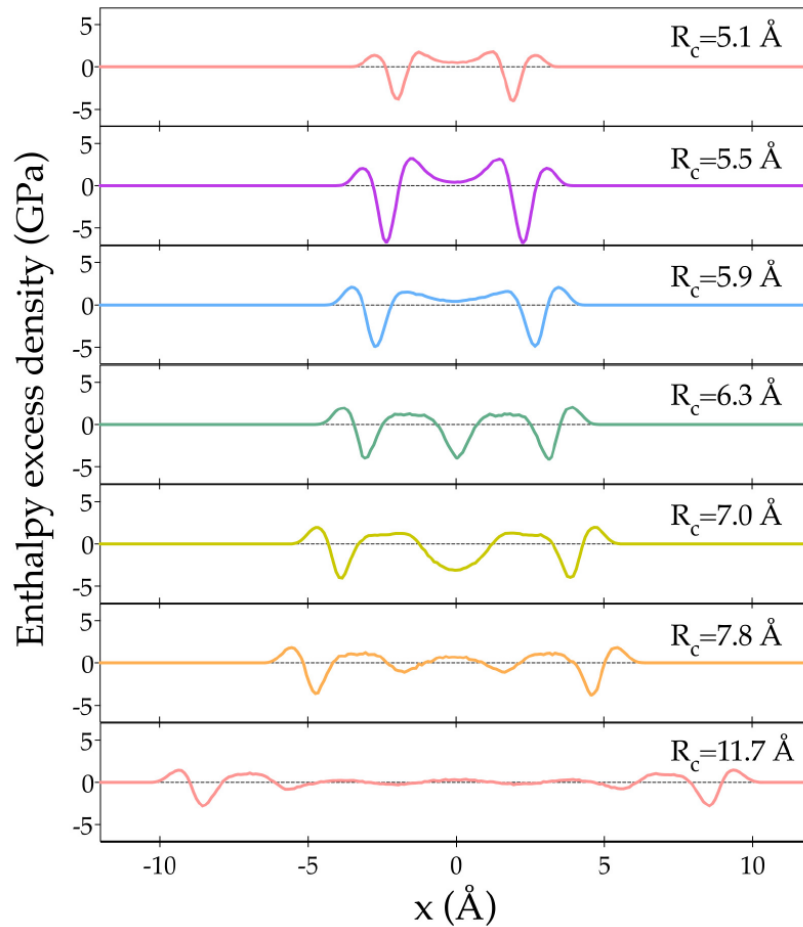
- Crucial role of hydrodynamics
- The MD-validated model can predict velocity in experimental situations

## Giant velocities, despite entrance effects and thermal short-circuit mechanism

- Reaching a plateau  $\sim 0.5$  m/s for large radii
- Complex dependency on  $R$  for small radii

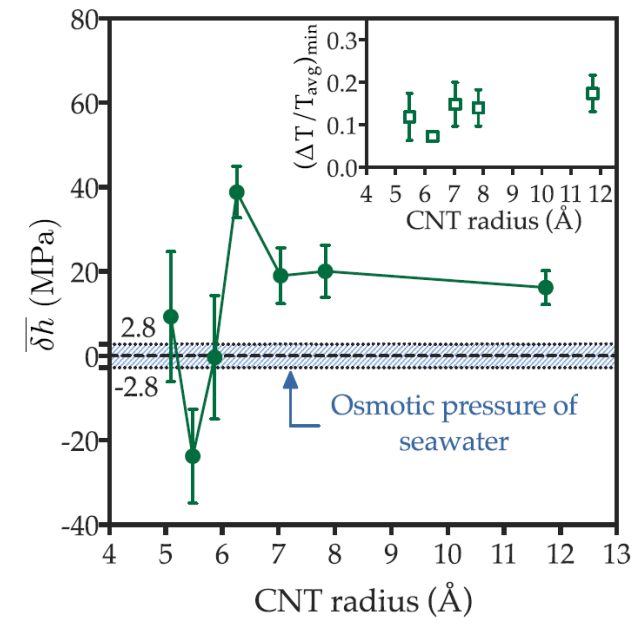


# Enthalpy profiles



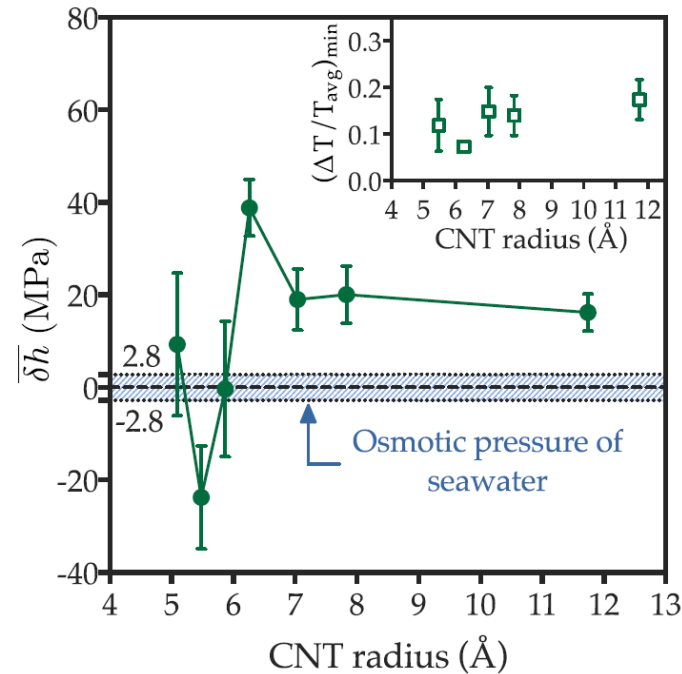
$$\text{Model } (\Delta II = 0): \quad U = \frac{R \bar{\delta h}}{2\lambda L + \pi C \eta} \times \frac{\Delta T}{T_{\text{avg}}}$$

$$\bar{\delta h} = \frac{1}{\pi R^2} \int_0^R dr 2\pi r \delta h_{\text{eq}}(r; T_{\text{avg}}),$$



- Weak confinement: model predicts  $\bar{\delta h}$  varies as  $1/R$ . → **constant velocity**
- Strong confinement: overlap of the interaction layers → complex evolution of  $\bar{\delta h}$

$$\Delta\Pi_{\max} = \overline{\delta h} \times \Delta T / T_{\text{avg}}$$



➤ **Desalination possible!**

➤ MD simulation with  $\Delta\Pi = 2.8 \text{ MPa} \rightarrow U = 0.38 \pm 0.18 \text{ m/s}$

(model:  $U = 0.44 \pm 0.06 \text{ m/s}$ )